Supplementary Information

Dynamics of a Hes1-Dll1 regulatory network in coupled muscle stem cells: stability, bifurcations, and coexistence of oscillatory states

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Zsófia Bujtár[a,b], Björn Goldenbogen[a], Jana Wolf[a,b*]

a) Max Delbrück Center for Molecular Medicine in the Helmholtz Association, Mathematical Modelling of Cellular Processes, Berlin, Germany

b) Freie Universität Berlin, Department of Mathematics and Computer Science, Berlin, Germany

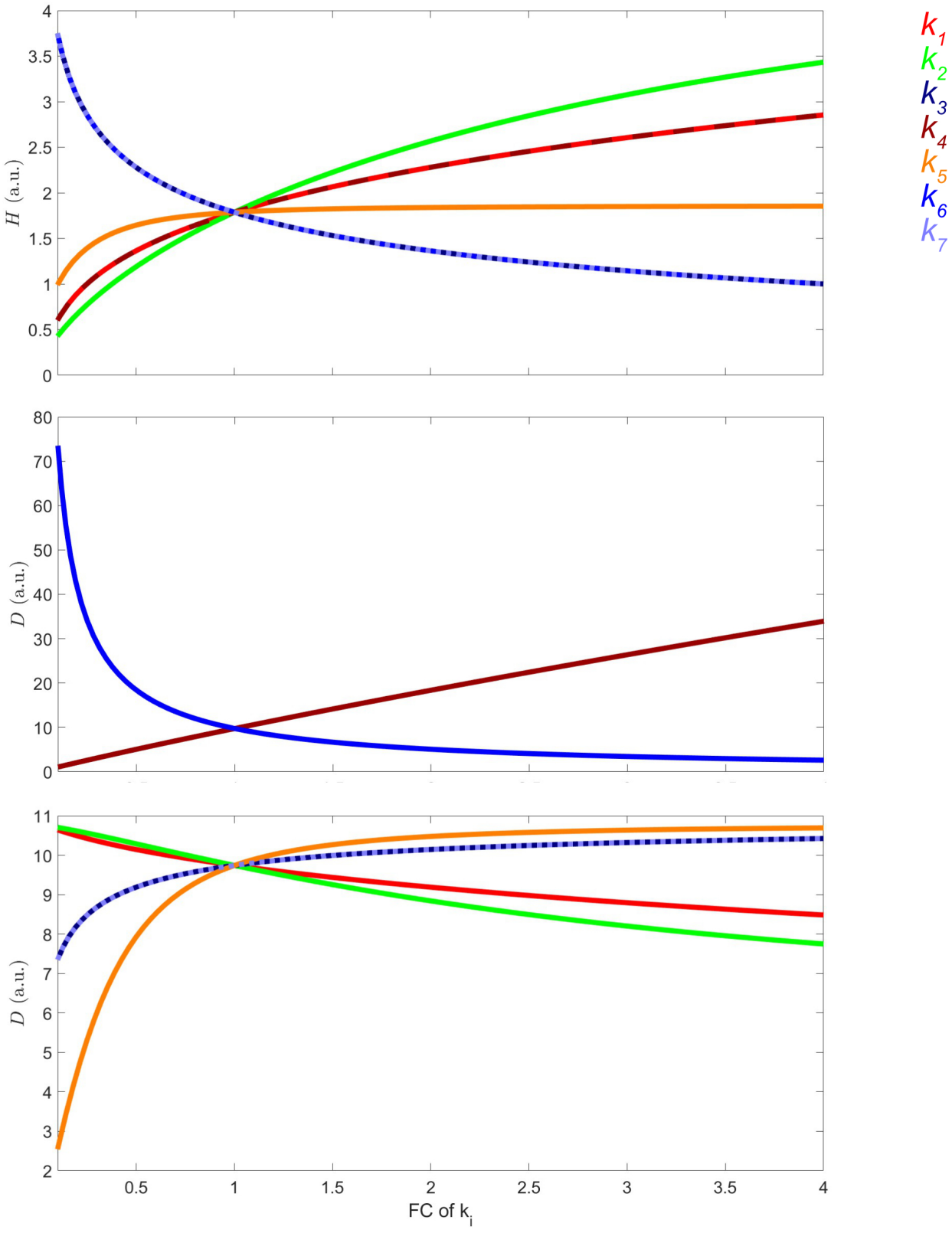
*corresponding author, e-mail address: jana.wolf@mdc-berlin.de

Parameter	Reference value	Unit
k_1	100.0	1/h
k_2	1.0	-
k_3	2.6	1/h
k_4	11.2	1/h
k_5	5.5	-
k_6	1.04	1/h
k_7	50.0	-
τ_1	0.725	h
τ_{21}	0.35	h
τ_{22}	1.3	h

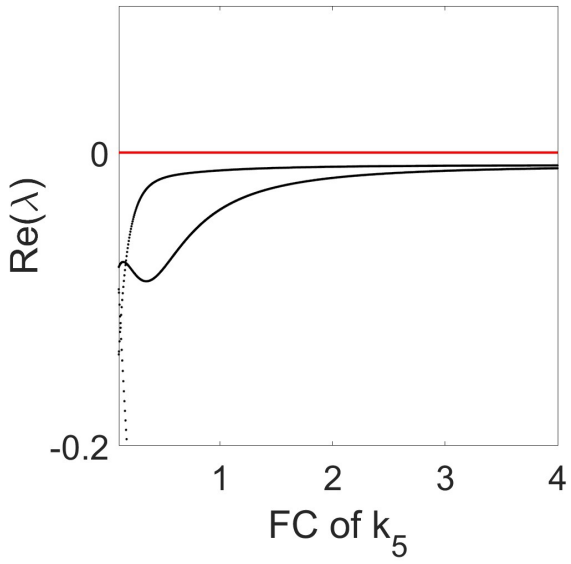
Supplementary Table 1 Reference parameter values of the two-coupled-cell model (Zhang et al. 2021). They are positive and real numbers without or with unit of time or its reciprocal.

Parameter	min FC	max FC	max/min
τ_1	0.98	1.05	1.07
k_3	0.96	1.08	1.13
k_7	0.88	1.06	1.20
k_2	0.89	1.08	1.21
k_1	0.94	1.15	1.22
k_4	0.93	1.14	1.23
k_6	0.88	1.09	1.24
k_5	0.14	1.79	12.79

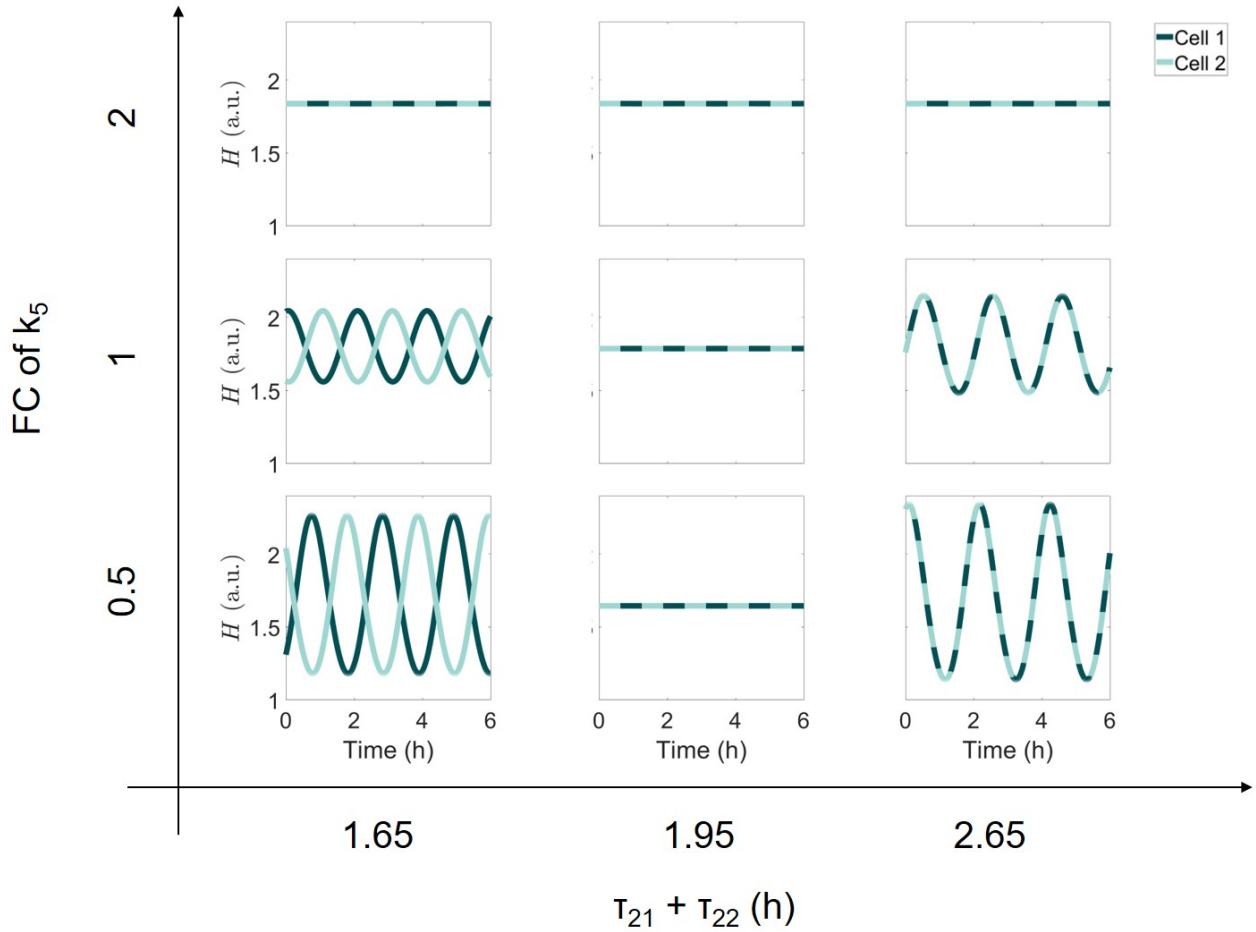
Supplementary Table 2 Estimated fold change ranges and ratios for $\mathbf{k_1}, ..., \mathbf{k_7}$ and τ_1 of the total coupling delay-dependent region around the reference parameter value.



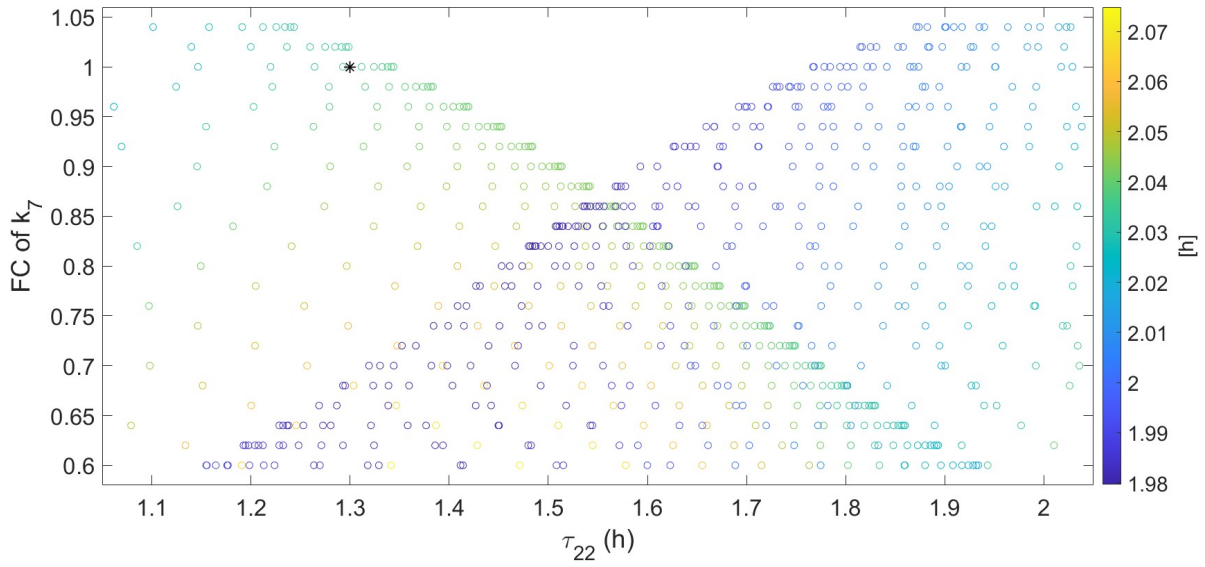
Supplementary Figure 1 Steady states values of H and D in both cells with respect to parameters $\mathbf{k_1}, ..., \mathbf{k_7}$. Parameters were varied individually between 0.1 FC and 4.0 FC.



Supplementary Figure 2 Variation of parameter k_5 in case of a total coupling delay ($\tau_{21} + \tau_{22}$) of 2 h shows only stable steady states. The eigenvalues with highest real part have only negative real parts. Parameter k_5 was varied from 0.1 FC to 4.0 FC.



Supplementary Figure 3 Simulations show oscillatory and sustained dynamics with respect to the total coupling delay $\tau_{21} + \tau_{22}$ and the parameter k_5.



Supplementary Figure 4 Impact of intercellular coupling delay and inverse of coupling strength on period of different periodic orbits. Point of reference is at $\tau_{22} = 1.3$ h and $k_7 = 1$ FC marked by a star.